Hafta 4
Bayes Teoremi

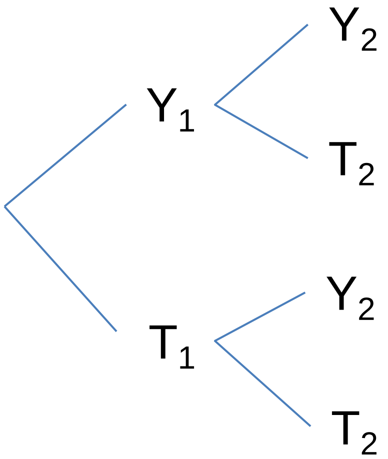
Hafta 3 Tekrar Sorusu

Bağımsız ve Bağdaşmaz (Ayrık) Olaylar

Örnek: Yazı-Tura deneyi iki kez tekrarlanıyor. $P(Y) = \frac{1}{3}$ $(hileli\ para)$ olduğuna göre en az bir tura gelme olasılığı nedir?

$$P(\Omega) = P\{Y, T\} = 1$$

$$P(Y) = \frac{1}{3}, \qquad P(T) = 1 - \frac{1}{3} = \frac{2}{3}$$



$$Y_1 \cap Y_2 = Y_1 Y_2$$

$$Y_1 \cap T_2 = Y_1 T_2$$

$$T_1 \cap Y_2 = T_1 Y_2$$

$$T_1 \cap T_2 = T_1 T_2$$

$$A = \{Y_1 T_2, T_1 Y_2, T_1 T_2\} = P(A) \neq \frac{|A|}{|\Omega|} \text{(eşit olasılığa sahip olmadığından)}$$

Bağımsız ve Bağdaşmaz (Ayrık) Olaylar

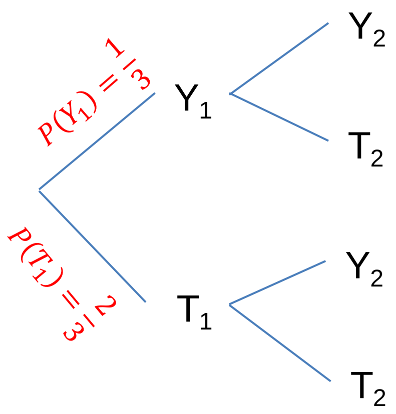
$A = \{Y_1T_2, T_1Y_2, T_1T_2\} = P(A) \neq \frac{|A|}{|\Omega|}$ (eşit olasılığa sahip olmadığından)

$A = \{Y_1T_2, T_1Y_2, T_1T_2\} = P(A) = P\{Y_1T_2, T_1Y_2, T_1T_2\}$

$P(A) = P\{Y_1T_2\} + P\{T_1Y_2\} + P\{T_1T_2\}$

? ? ?

Bağımsız ve Bağdaşmaz (Ayrık) Olaylar



$Y_1 \cap Y_2 = Y_1Y_2$

$Y_1 \cap T_2 = Y_1T_2$

$T_1 \cap Y_2 = T_1Y_2$

$T_1 \cap T_2 = T_1T_2$

$$P(A) = P\{Y_1T_2\} + P\{T_1Y_2\} + P\{T_1T_2\}$$

? ? ?

$$P(Y_1 \cap Y_2) = P(Y_1Y_2) = \underbrace{P(Y_1)}_{\frac{1}{3}}.\underbrace{P(Y_2|Y_1)}_{P(Y_2) = \frac{1}{3}}$$

$$P(Y_1Y_2) = P(Y_1).P(Y_2)$$

(Bağımsız Olay)

$$P(Y_1Y_2) = \frac{1}{3} x \frac{1}{3} = \frac{1}{9}$$

Bağımsız ve Bağdaşmaz (Ayrık) Olaylar

$$\Omega = \{\underbrace{Y_1Y_2}_{\frac{1}{9}}, \underbrace{Y_1T_2}_{\frac{2}{9}}, \underbrace{T_1Y_2}_{\frac{2}{9}}, \underbrace{T_1T_2}_{\frac{4}{9}}\}$$

$$P(Y_1T_2) = \frac{1}{3} x \frac{2}{3} = \frac{2}{9}$$

$$A = \{Y_1T_2, T_1Y_2, T_1T_2\} = P(A) \neq \frac{|A|}{|\Omega|} \text{(eşit olasılığa sahip olmadığından)}$$

$$P(A) = P\{Y_1T_2\} + P\{T_1Y_2\} + P\{T_1T_2\} = \frac{2}{9} + \frac{2}{9} + \frac{4}{9} = \frac{8}{9}$$

Bayes Teoremi - Sezgisel

Bayes Teoremi: Sezgisel

1,000,000 insan

1 / 10,000 insan

99% Doğruluk

Test Sonucu Hasta

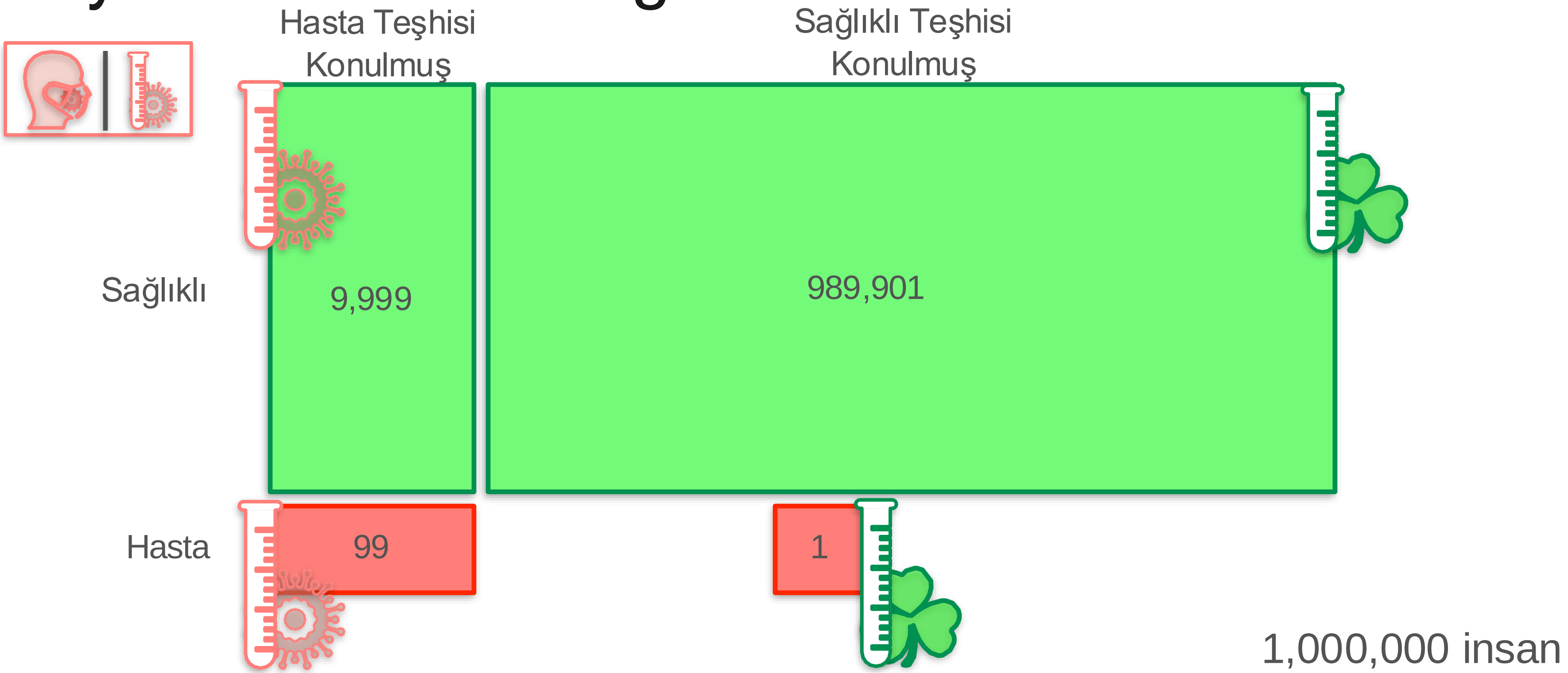
	Hasta Tespit Edildi	Sağlıklı Tespit Edildi
100 insan	99	1
100 insan	1	99

Test sonucunuza göre hasta olduğunuz VERİLDİĞİNDE hasta olma olasılığı?

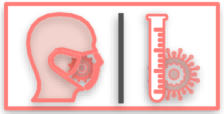
Bayes Teoremi: Sezgisel

1,000,000 insan

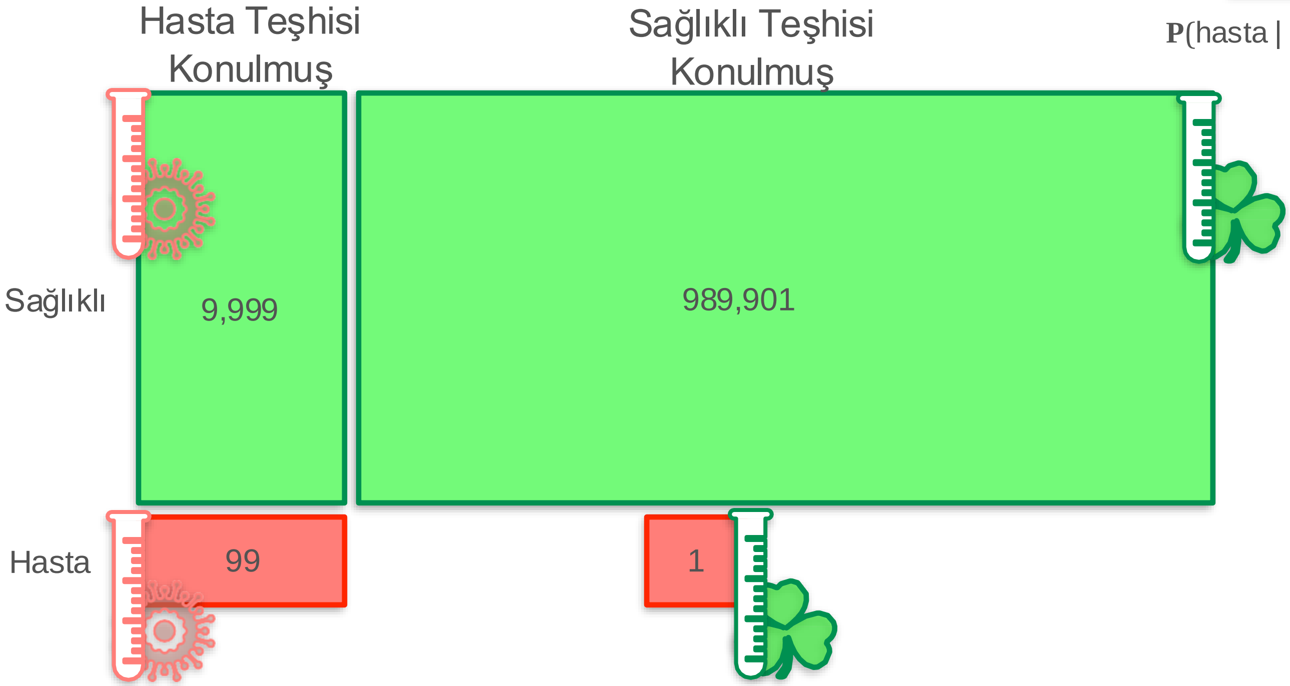
Bayes Teoremi: Sezgisel



Bayes Teoremi: Sezgisel

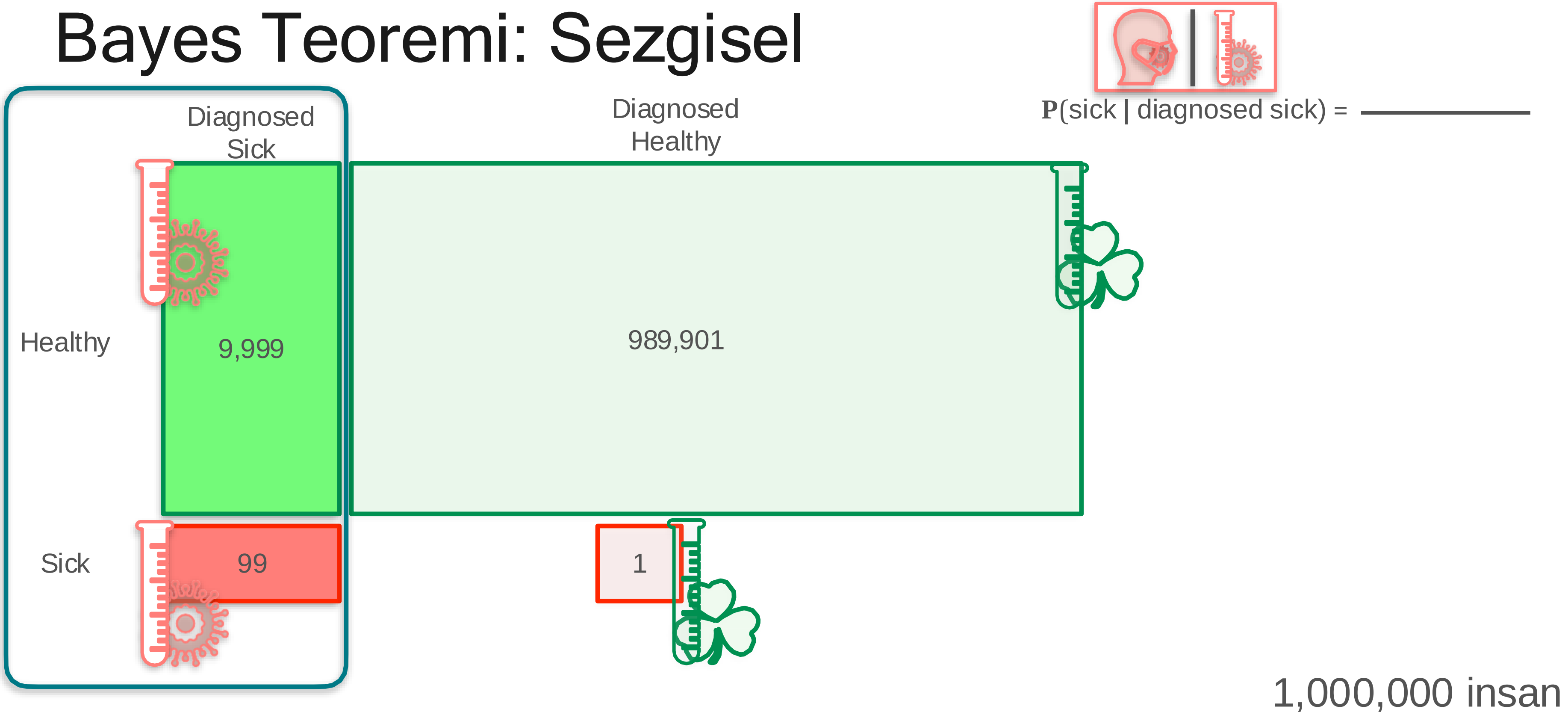


P(hasta | hasta teşhis edildi) = ________

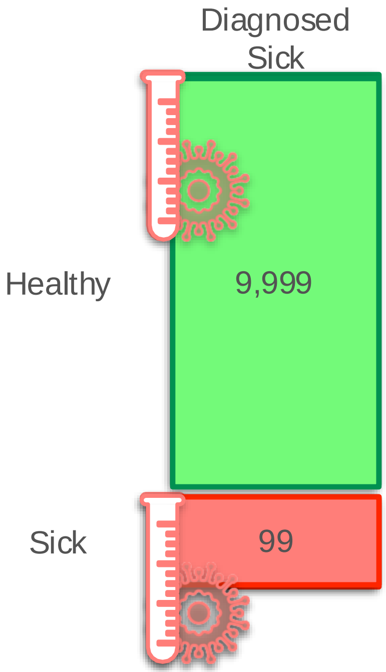


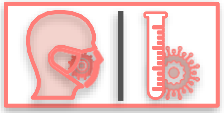
1,000,000 insan

Bayes Teoremi: Sezgisel

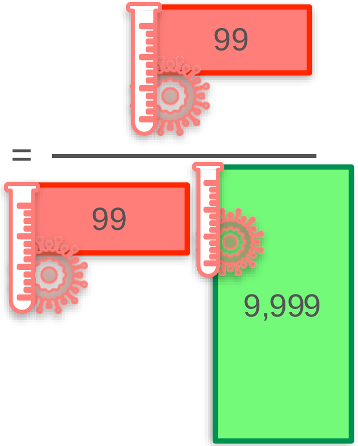


Bayes Teoremi: Sezgisel





$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) =$

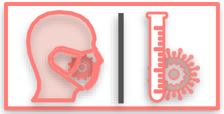


$$= \frac{99}{99 + 9999}$$

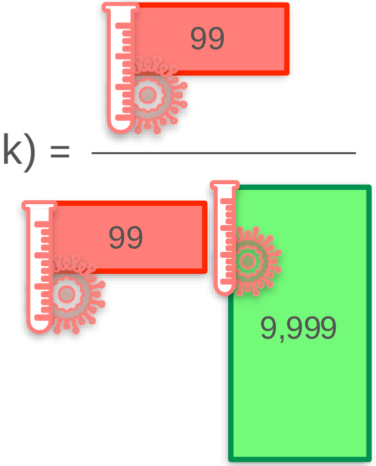
$$= \frac{99}{10098} = 0.0098$$

$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

Bayes Teoremi: Sezgisel



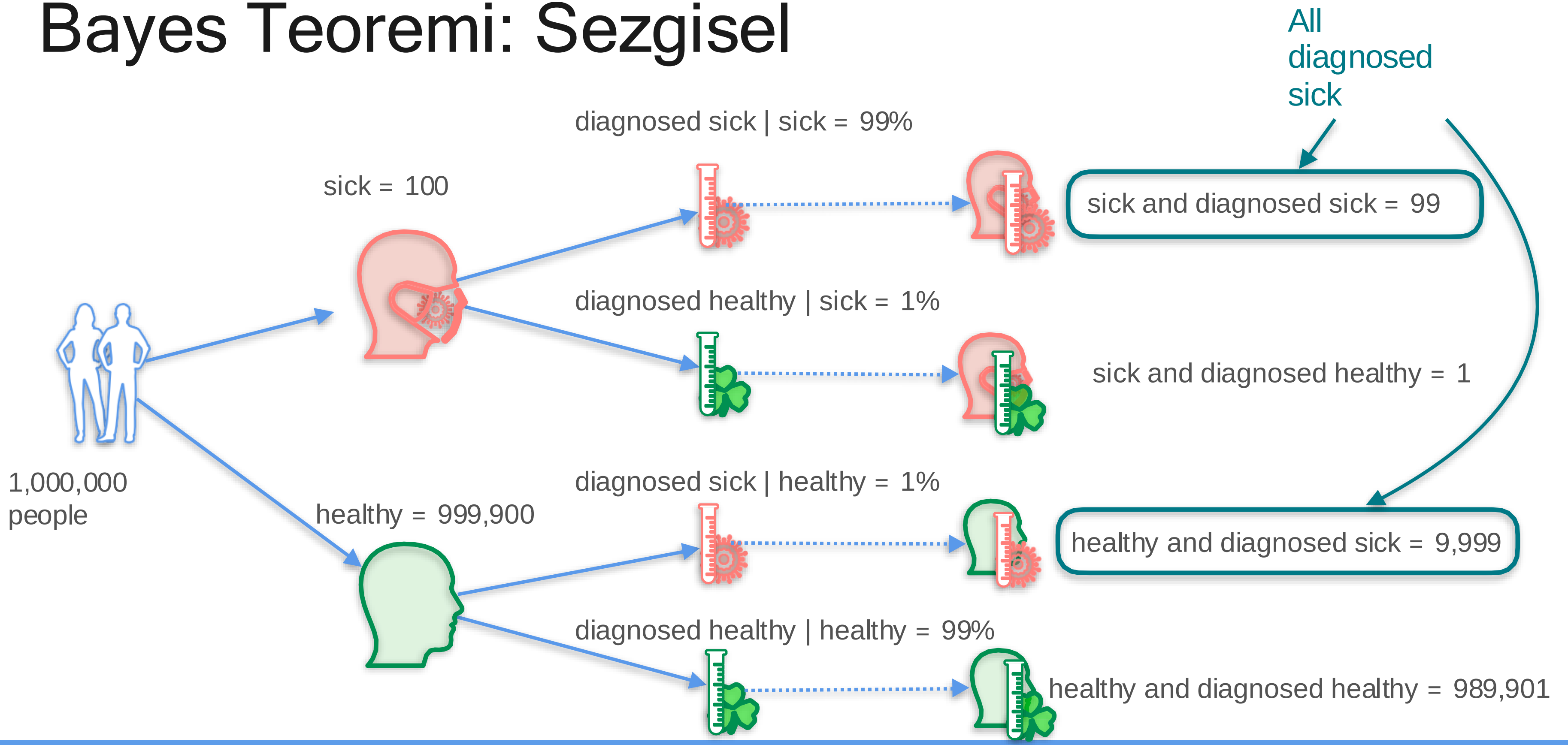
P(sick | diagnosed sick) =



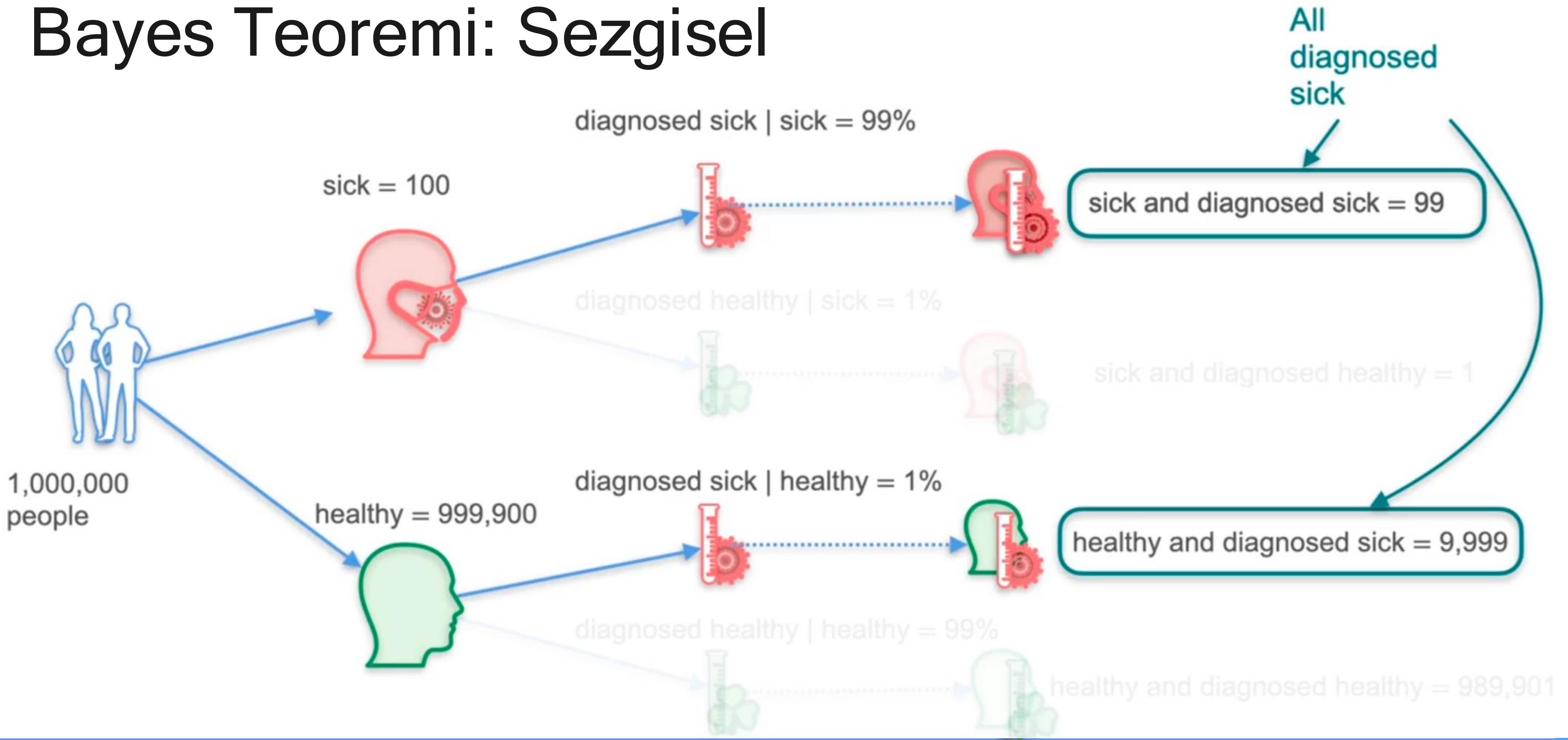
$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

$$= \frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$$

Bayes Teoremi: Sezgisel

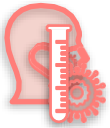


Bayes Teoremi: Sezgisel



Bayes Teoremi: Sezgisel

P(sick | diagnosed sick) =



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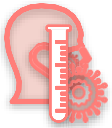
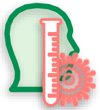
 + 

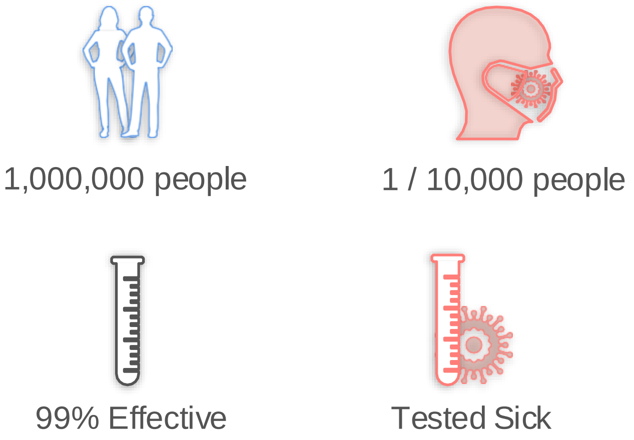
$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9{,}999 + \text{sick and diagnosed sick} = 99}$$

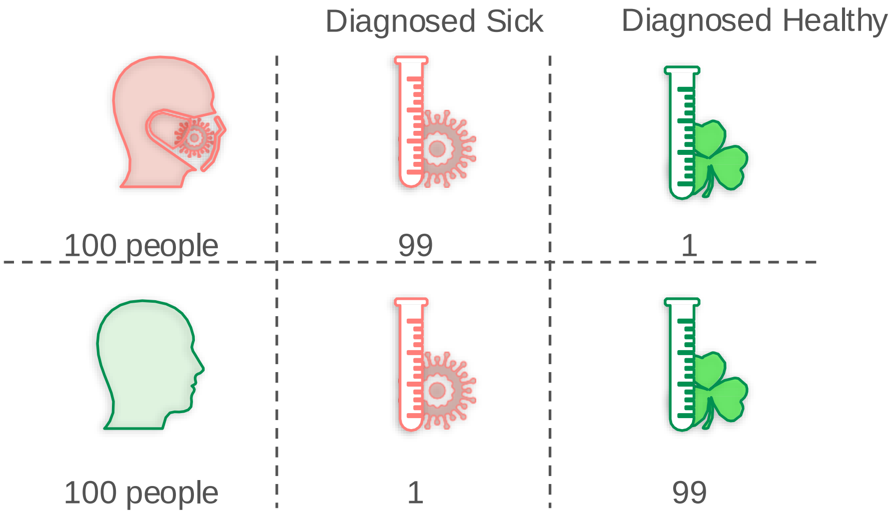
$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098} = 0.0098$$

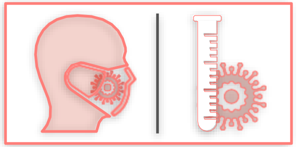
Bayes Teorem – Matematiksel Formül

DeepLearning.AI

Bayes Teorem: Formül

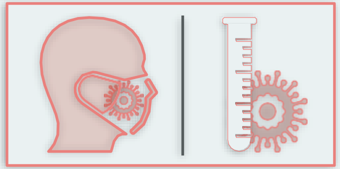






Test sonucunuzun pozitif olduğu **VERİLDİĞİNDE hasta olma** olasılığınız nedir

Bayes Teorem: Formül



$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

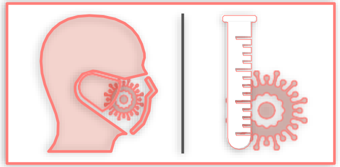
$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

teste göre

Test sonucunuzun pozitif olduğu **VERİLDİĞİNDE hasta olma** olasılığınız nedir

Bayes Teorem: Formül



$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

A : sick (hasta)

B : diagnosed sick (hasta teşhisi konmuş)

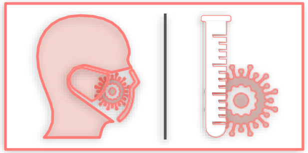
$\mathbf{P}(A \mid B) = ?$

From Conditional Probability

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(\text{sick and diagnosed sick})}{\mathbf{P}(\text{diagnosed sick})}$$

Bayes Teorem: Formül



A : sick

B : diagnosed sick

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

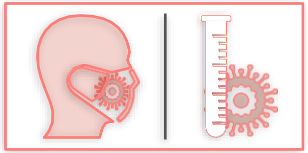
$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(\text{sick and diagnosed sick})}{\mathbf{P}(\text{diagnosed sick})}$$

$\mathbf{P}(\text{sick and diagnosed sick}) = ?$

$\mathbf{P}(\text{diagnosed sick}) = ?$

BAYES THEOREM FORMULA CAN HELP

Bayes Teorem: Formül



A : sick

B : diagnosed sick

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$\mathbf{P}(A \mid B) = \mathbf{P}(\text{sick} \mid \text{diagnosed sick})$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(\text{sick and diagnosed sick}) = ?}{\mathbf{P}(\text{diagnosed sick}) = ?}$$

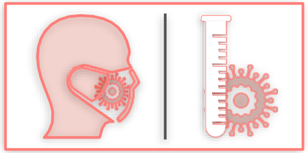
From Conditional Probability

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B \mid A)$$

$$\mathbf{P}(\text{sick and diagnosed sick}) = \mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick})$$

DeepLearning.AI

Bayes Teorem: Formül



A : sick

B : diagnosed sick

$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$$

$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

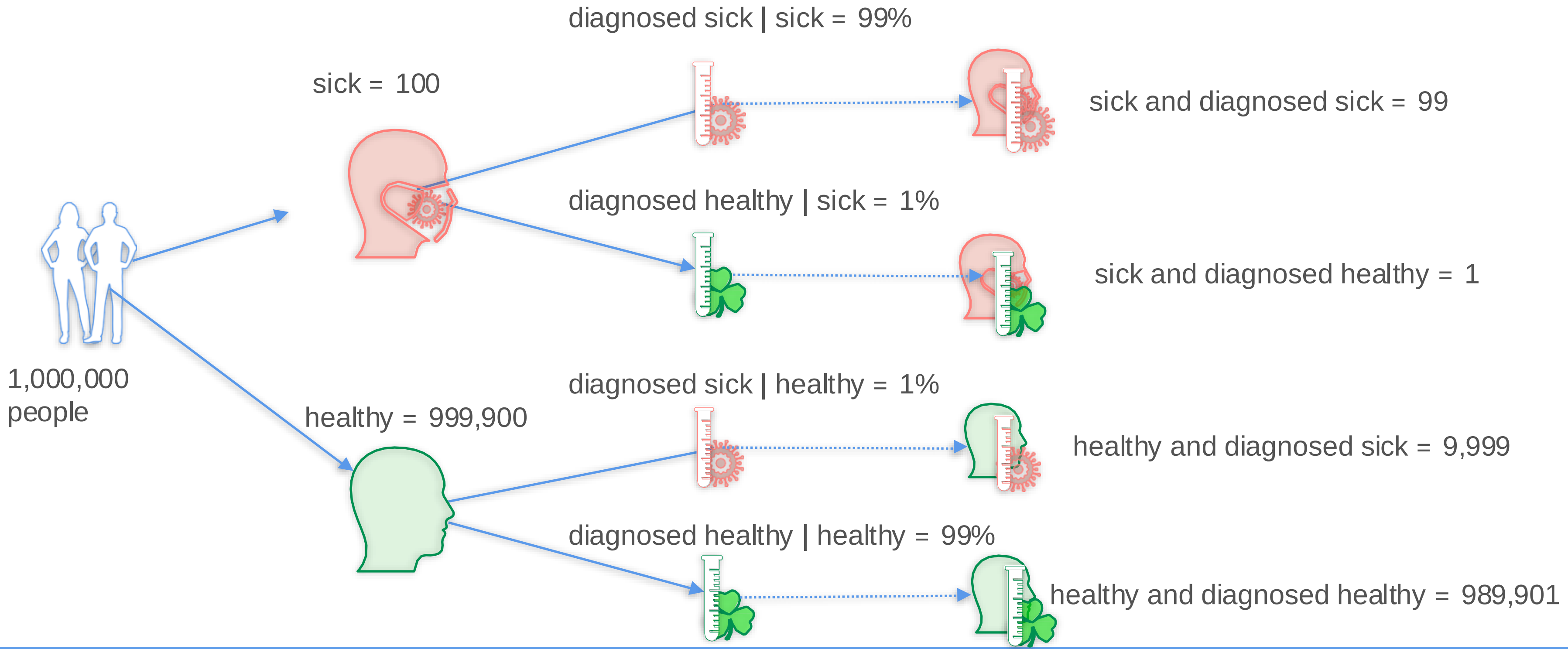
$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

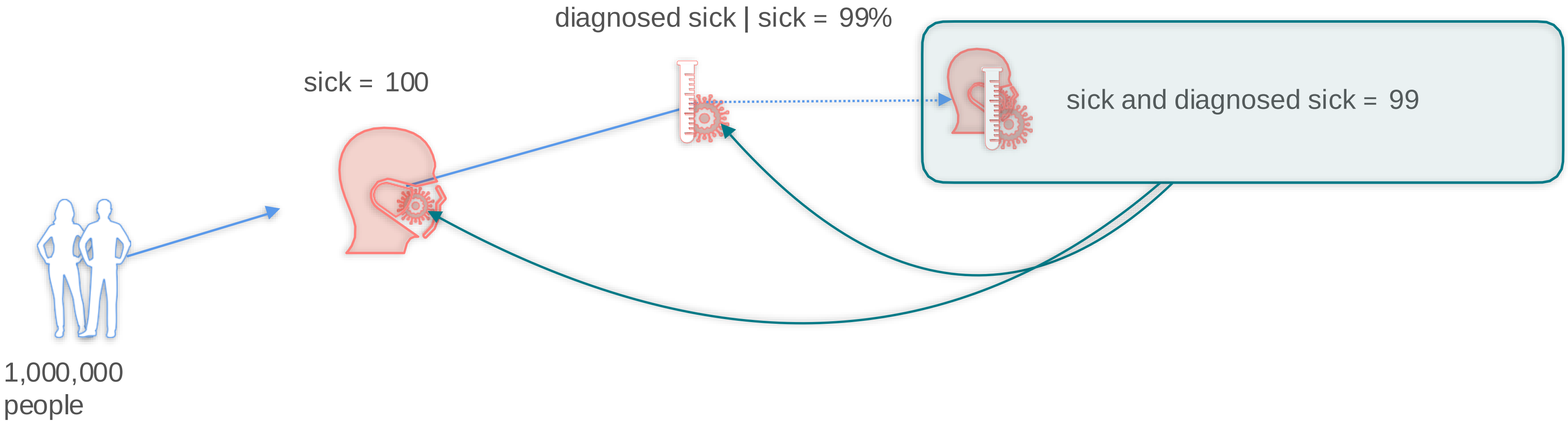
$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick})}{\mathbf{P}(\text{diagnosed sick}) = ?}$$

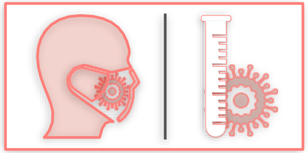
Bayes Teorem: Formül



Bayes Teorem: Formül



Bayes Teorem: Formül



A : sick

B : diagnosed sick

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \ ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

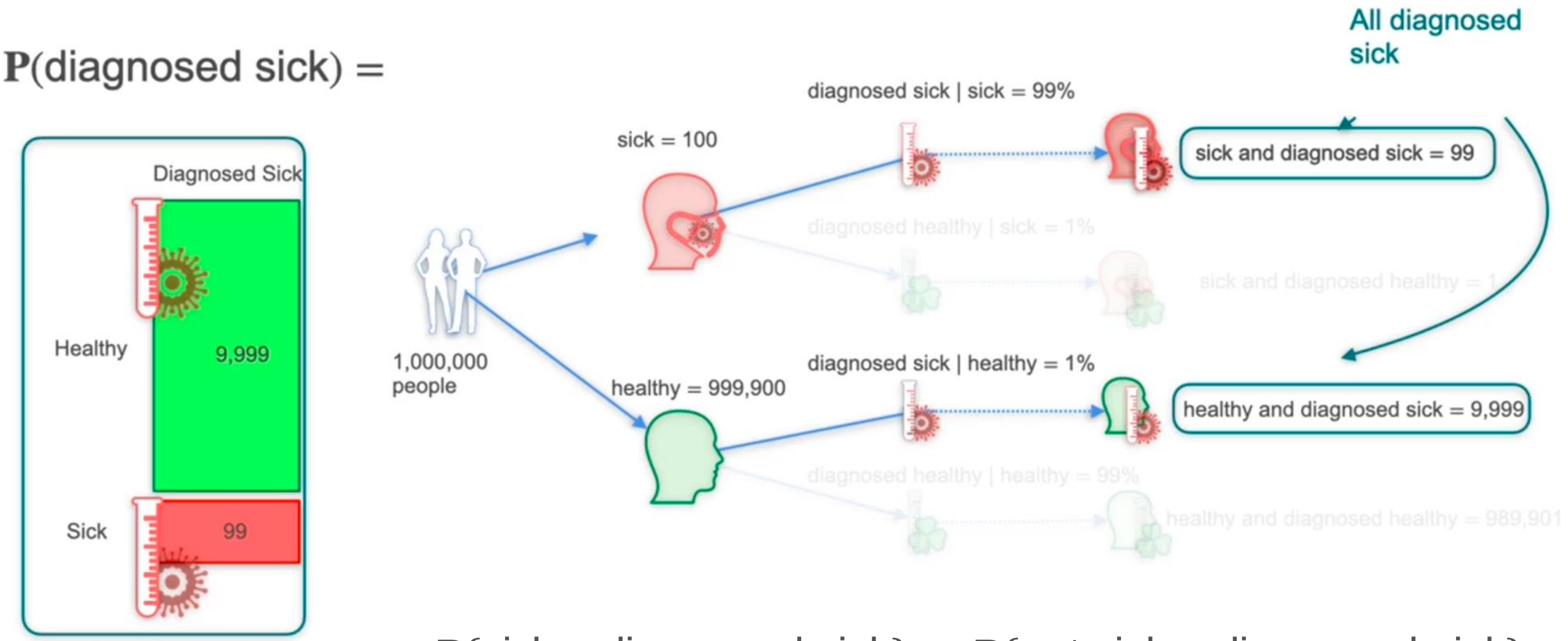
$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick})}{\mathbf{P}(\text{diagnosed sick}) = \ ?}$$

$\mathbf{P}(\text{diagnosed sick}) =$

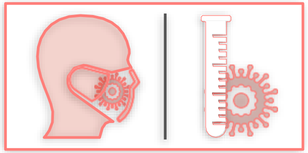
Bayes Teorem: Formül

$\mathbf{P}(\text{diagnosed sick}) =$



$\mathbf{P}(\text{sick} \cap \text{diagnosed sick}) + \mathbf{P}(\text{not sick} \cap \text{diagnosed sick})$

Bayes Teorem: Formül



A : sick

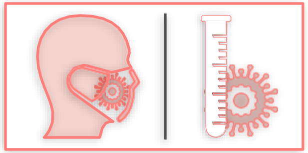
B : diagnosed sick

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick})}{\mathbf{P}(\text{diagnosed sick}) = \ ?}$$

$$\mathbf{P}(\text{diagnosed sick}) = \mathbf{P}(\text{sick} \cap \text{diagnosed sick}) + \mathbf{P}(\text{not sick} \cap \text{diagnosed sick})$$

Bayes Teorem: Formül



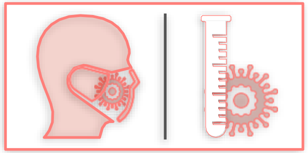
A : sick

B : diagnosed sick

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick})}{\mathbf{P}(\text{sick} \cap \text{diagnosed sick}) + \mathbf{P}(\text{not sick} \cap \text{diagnosed sick})}$$

Bayes Teorem: Formül



A : sick

B : diagnosed sick

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick})}{\mathbf{P}(\text{sick} \cap \text{diagnosed sick}) + \mathbf{P}(\text{not sick} \cap \text{diagnosed sick})}$$

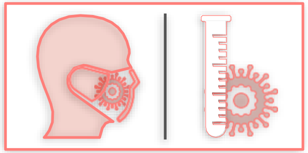
$$\mathbf{P}(\text{sick} \cap \text{diagnosed sick}) = \mathbf{P}(A \cap B)$$

$$= \mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick})$$

$$\mathbf{P}(\text{not sick} \cap \text{diagnosed sick}) = \mathbf{P}(A' \cap B)$$

$$= \mathbf{P}(\text{not sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{not sick})$$

Bayes Teorem: Formül



A : sick

B : diagnosed sick

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} \quad ?$$

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick})}{\mathbf{P}(\text{sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{sick}) + \mathbf{P}(\text{not sick}) \cdot \mathbf{P}(\text{diagnosed sick} \mid \text{not sick})}$$

$\mathbf{P}(\text{sick}) = 0.01\%$

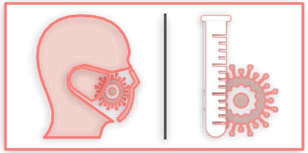
$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

BAYES THEOREM FORMULA

Bayes Teorem: Formül



A : sick

B : diagnosed sick

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B|A)}{\mathbf{P}(A) \cdot \mathbf{P}(B|A) + \mathbf{P}(A') \cdot \mathbf{P}(B|A')}$$

$\mathbf{P}(A) = 0.01\%$

$\mathbf{P}(A') = 99.99\%$

$\mathbf{P}(B|A) = 99\%$

$\mathbf{P}(B|A') = 1\%$

BAYES THEOREM FORMULA

Bayes Teorem: Formül

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B|A)}{\mathbf{P}(A) \cdot \mathbf{P}(B|A) + \mathbf{P}(A') \cdot \mathbf{P}(B|A')}$$

$\mathbf{P}(A)$ = 0.01%

$\mathbf{P}(A')$ = 99.99%

$\mathbf{P}(B|A)$ = 99%

$\mathbf{P}(B|A')$ = 1%

Bayes Teorem: Formül

$$P(A|B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} \mid \text{not sick})}$$

P(sick) = 0.01%

P(not sick) = 99.99%

P(diagnosed sick | sick) = 99%

P(diagnosed sick | not sick) = 1%

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(A') \cdot P(B|A')}$$

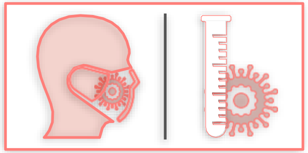
$P(A) = 0.01\%$

$P(A') = 99.99\%$

$P(B|A) = 99\%$

$P(B|A') = 1\%$

Bayes Teorem: Formül



A : sick

B : diagnosed sick

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B|A)}{\mathbf{P}(A) \cdot \mathbf{P}(B|A) + \mathbf{P}(A') \cdot \mathbf{P}(B|A')}$$

$\mathbf{P}(A)$ = 0.01%

$\mathbf{P}(A')$ = 99.99%

$\mathbf{P}(B|A)$ = 99%

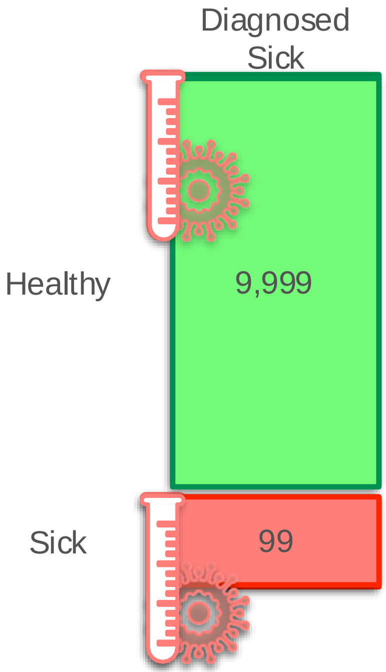
$\mathbf{P}(B|A')$ = 1%

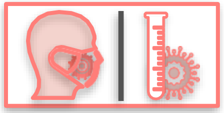
$$\mathbf{P}(A|B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

$$\mathbf{P}(A|B) = 0.0098$$

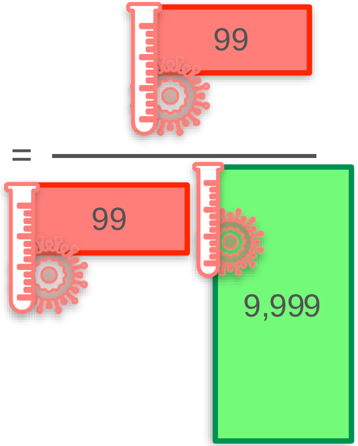
BAYES THEOREM FORMULA

(Hatırlatma) Bayes Teoremi: Sezgisel





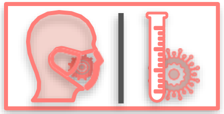
P(sick | diagnosed sick) =



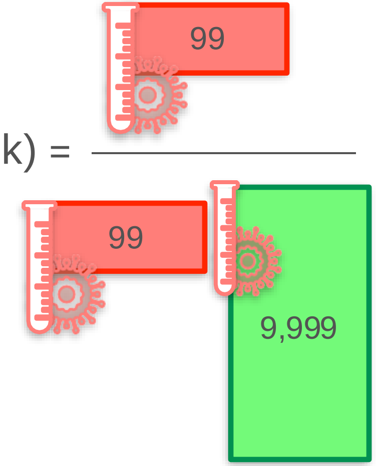
$$= \frac{99}{99 + 9999} = \frac{99}{10098} = 0.0098$$

$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

Bayes Teoremi: Sezgisel



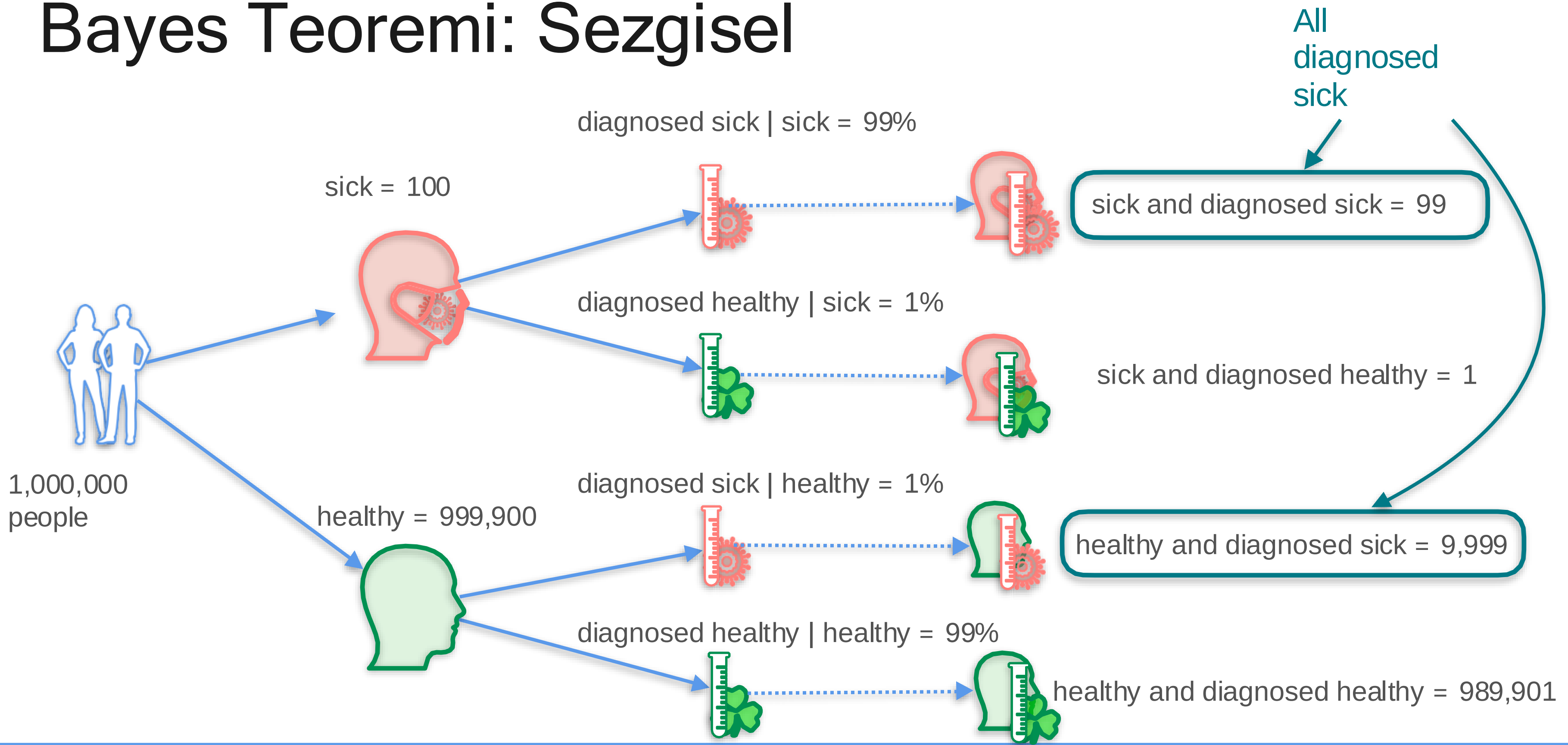
P(sick | diagnosed sick) =



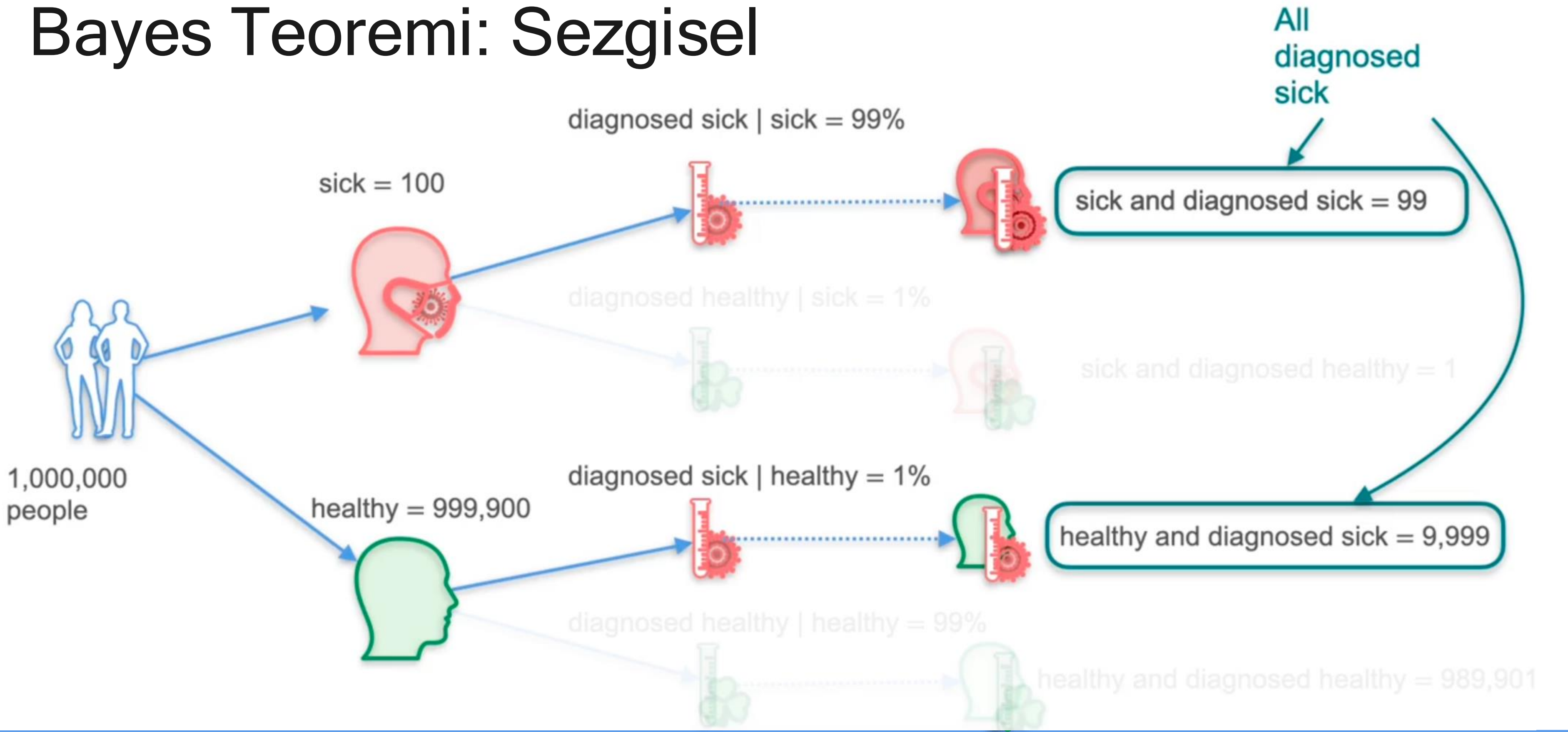
$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

$$= \frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$$

Bayes Teoremi: Sezgisel

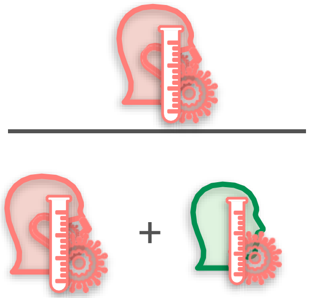


Bayes Teoremi: Sezgisel



Bayes Teoremi: Sezgisel

$\mathbf{P}$(sick | diagnosed sick) =



$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9{,}999 + \text{sick and diagnosed sick} = 99}$$

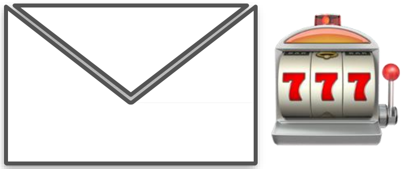
$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098} = 0.0098$$

Bayes Teorem - Spam Örneği

Bayes Teorem: Spam Örneği

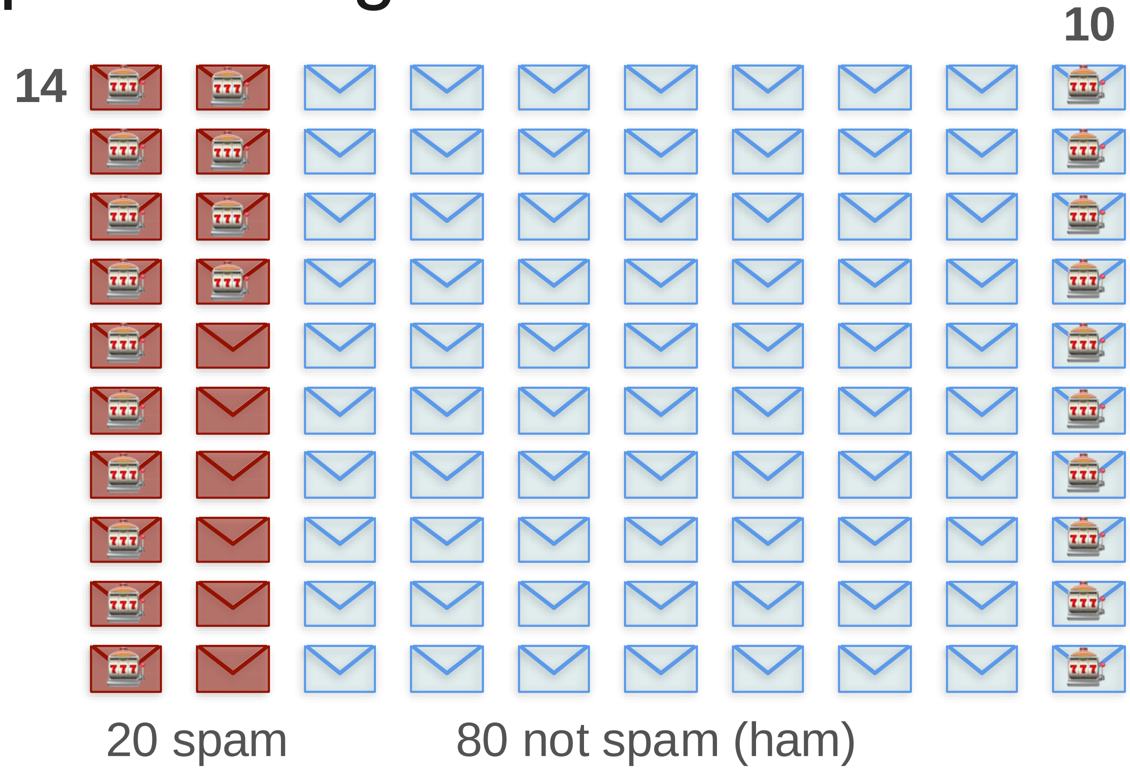
Bayes Teorem: Spam Örneği

Bayes Teorem: Spam Örneği

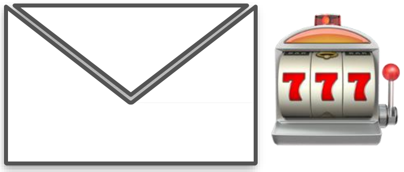


Piyango (lottery) kelimesi içeren bir e-postanın spam olma olasılığı nedir?

$\mathbf{P}$(spam | lottery)



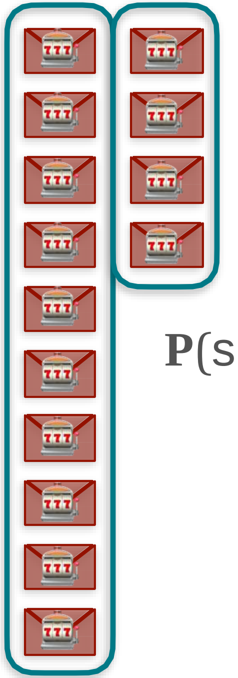
Bayes Teorem: Spam Örneği (Sezgisel Çözüm)



$\mathbf{P}(\text{spam} \mid \text{lottery})$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

A: spam
B: lottery



14

24 emails

containing lottery

(piyango içeriyor)

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

$$= \frac{7}{12} = 0.583$$

10

Bayes Teorem: Spam Örneği (Formülsel Çözüm)

$$\mathbf{P}(\text{spam} \mid \text{lottery})$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A)}{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A) + \mathbf{P}(A') \cdot \mathbf{P}(B \mid A')}$$

A: Email is spam

B: Email contains lottery

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) + \mathbf{P}(\text{not spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{not spam})}$$

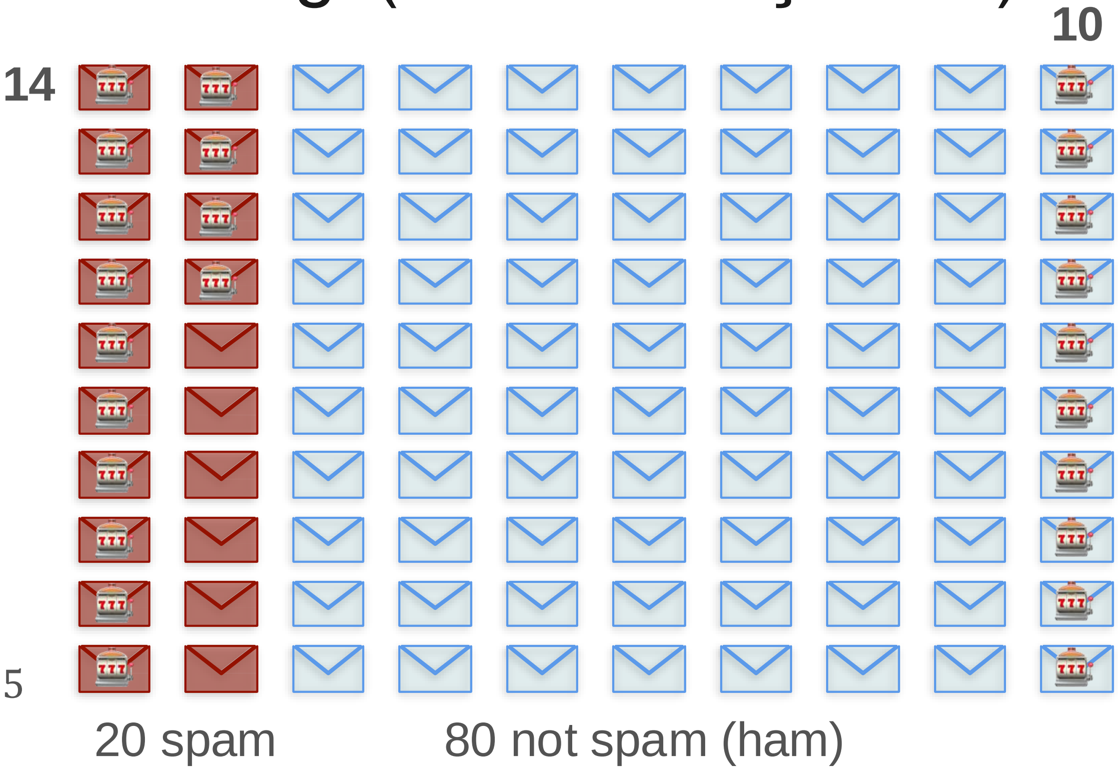
Bayes Teorem: Spam Örneği (Formülsel Çözüm)

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{not spam}) = \frac{80}{100} = 0.8$$

$$\mathbf{P}(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$

$$\mathbf{P}(\text{lottery} \mid \text{not spam}) = \frac{10}{80} = 0.125$$



Bayes Teorem: Spam Örneği (Formülsel Çözüm)

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{not spam}) = 0.8$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{lottery} \mid \text{not spam}) = 0.125$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) + \mathbf{P}(\text{not spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{not spam})}$$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)} = 0.583$$

Bayes Teorem: Spam Örneği (Formülsel Çözüm)

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{not spam}) = 0.8$

$\mathbf{P}(\text{lottery} \mid \text{not spam}) = 0.125$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) + \mathbf{P}(\text{not spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{not spam})}$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A)}{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A) + \mathbf{P}(A') \cdot \mathbf{P}(B \mid A')}$$

Bayes Teorem: Spam Örneği (Formülsel Çözüm)

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{not spam}) = 0.8$

$\mathbf{P}(\text{lottery} \mid \text{not spam}) = 0.125$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) + \mathbf{P}(\text{not spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{not spam})}$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A)}{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A) + \mathbf{P}(A') \cdot \mathbf{P}(B \mid A')}$$

$P(A \cap B)$

$P(B)$

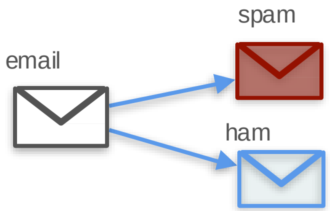
Bayes Teorem - Prior ve Posterior

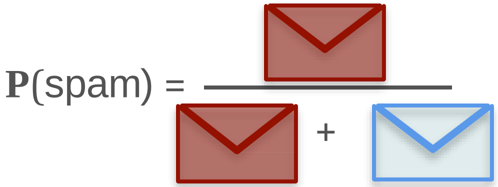
Bayes Teorem

PRIOR	EVENT	POSTERIOR
$\mathbf{P}(A)$	E	$\mathbf{P}(A \mid E)$

Prior ve Posterior

PRIOR

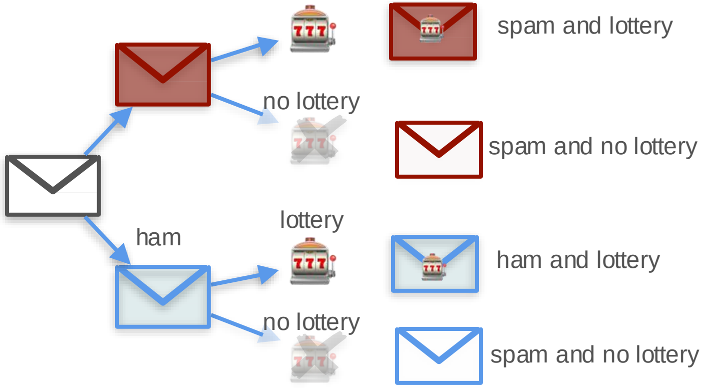


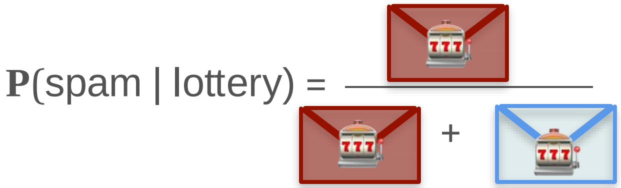


EVENT

Email contains lottery

POSTERIOR





Prior ve Posterior

Tıbbi örnekte, bir önceki (prior) ve bir sonraki (posterior) olasılık vardı. Önceki, ilk hasta olma olasılığıydı. Olay pozitif teşhis edildiği gerçeğidir. Sonraki ise, pozitif teşhis koyduğunuz göz önüne alındığında hasta olma olasılığınızdı.

Prior ve Posterior

olaydan sonra örnek uzay değişir

PRIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{sum} = 10) = \frac{3}{36}$$

EVENT

?

1st dice is 6

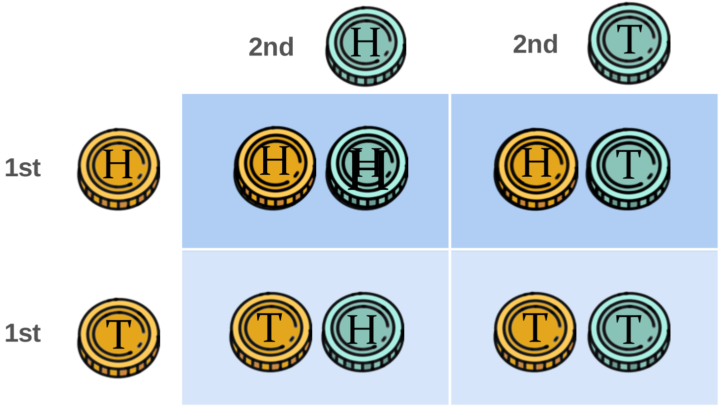
POSTERIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{sum} = 10 \mid \text{1st is 6}) = \frac{1}{6}$$

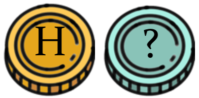
Prior ve Posterior

PRIOR



$$\mathbf{P}(HH) = \frac{1}{4}$$

EVENT



1st coin is

POSTERIOR

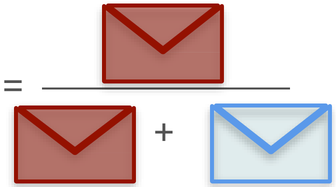
$$\mathbf{P}(HH \,|\, \text{1st is } H) = \frac{1}{2}$$

DeepLearning.AI

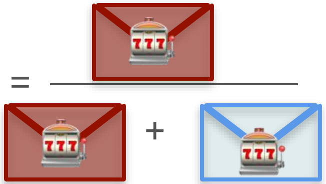
Bayes Teorem
Naive Bayes Model

2 Olay?

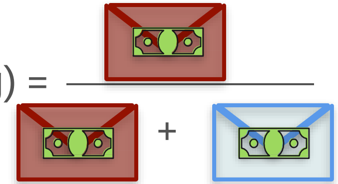
PRIOR | **EVENT** | **POSTERIOR**

P(spam) =



Email contains ‘lottery’

P(spam | lottery) =



Email contains ‘winning'

P(spam | winning) =



Email contains ‘lottery’ and ‘winning’

?

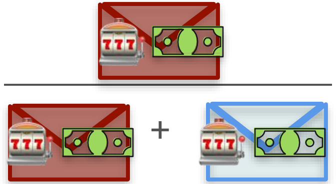
2 Olay?

EVENT

Email contains ‘lottery’ and ‘winning’

POSTERIOR

$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) =$



$$\frac{\text{\# Spam emails with 'lottery' and 'winning'}}{\text{\# Spam emails}}$$

?

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

2 Olaydan Fazla?

EVENT **POSTERIOR**

Email contains $w_1, w_2, \ldots, w_{100}$

$$\frac{\text{\# Spam emails with } w_1, \ldots, w_{100}}{\text{\#Spam emails}}$$ 0

?

$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_{100}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{ham})}$$

Olasılığı daha hızlı hesaplamanın bir yolu var mı?

Naive assumption

The appearances of ‘lottery’ and ‘winning’ are independent

(piyango ve kazanma durumlarının ortaya çıkmasının bağımsız olduğu varsayılır)

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

Olasılığı daha hızlı hesaplamanın bir yolu var mı?

Naive assumption

The appearances of ‘lottery’ and ‘winning’ are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery} \mid \text{ham}) \cdot \mathbf{P}(\text{winning} \mid \text{ham})}$$

Olasılığı daha hızlı hesaplamanın bir yolu var mı?

Naive assumption

The appearances of the words $w_1, w_2, \ldots, w_n$ are independent

$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_n) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1 \mid \text{spam}) \cdots \mathbf{P}(w_n \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1 \mid \text{spam}) \cdots \mathbf{P}(w_n \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1 \mid \text{ham}) \cdots \mathbf{P}(w_n \mid \text{ham})}$$

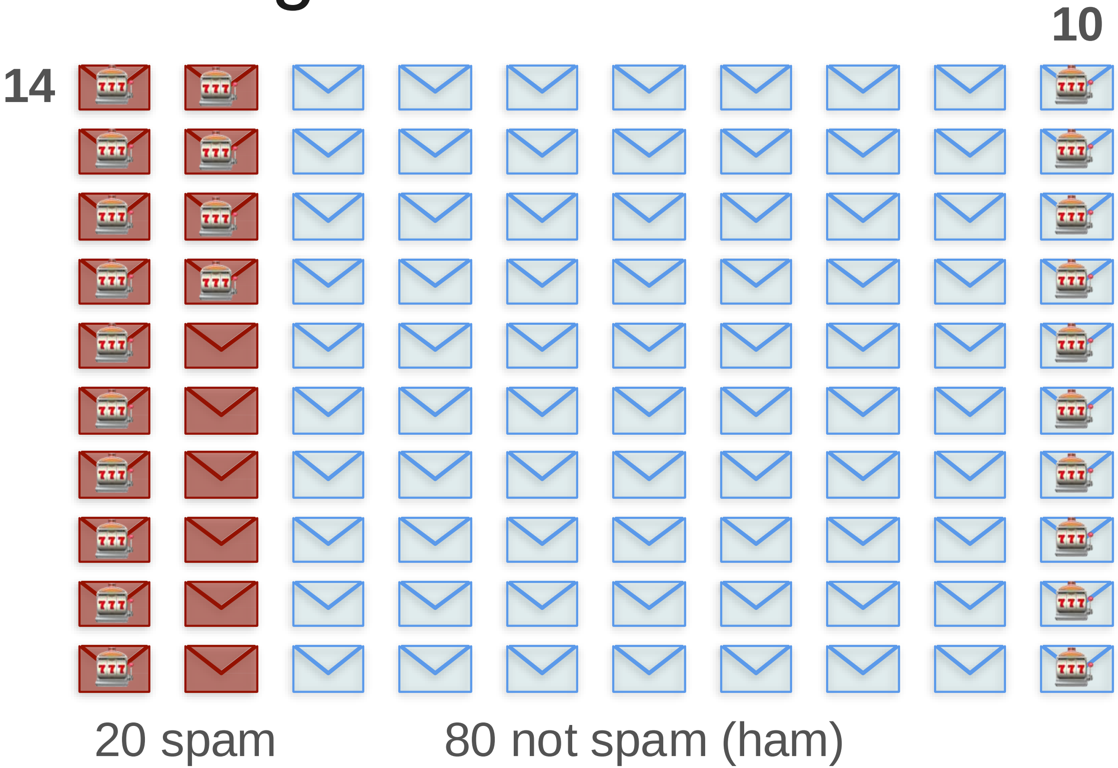
Naive Bayes: Spam Örneği

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$

$$P(\text{lottery} \mid \text{ham}) = \frac{10}{80} = 0.125$$



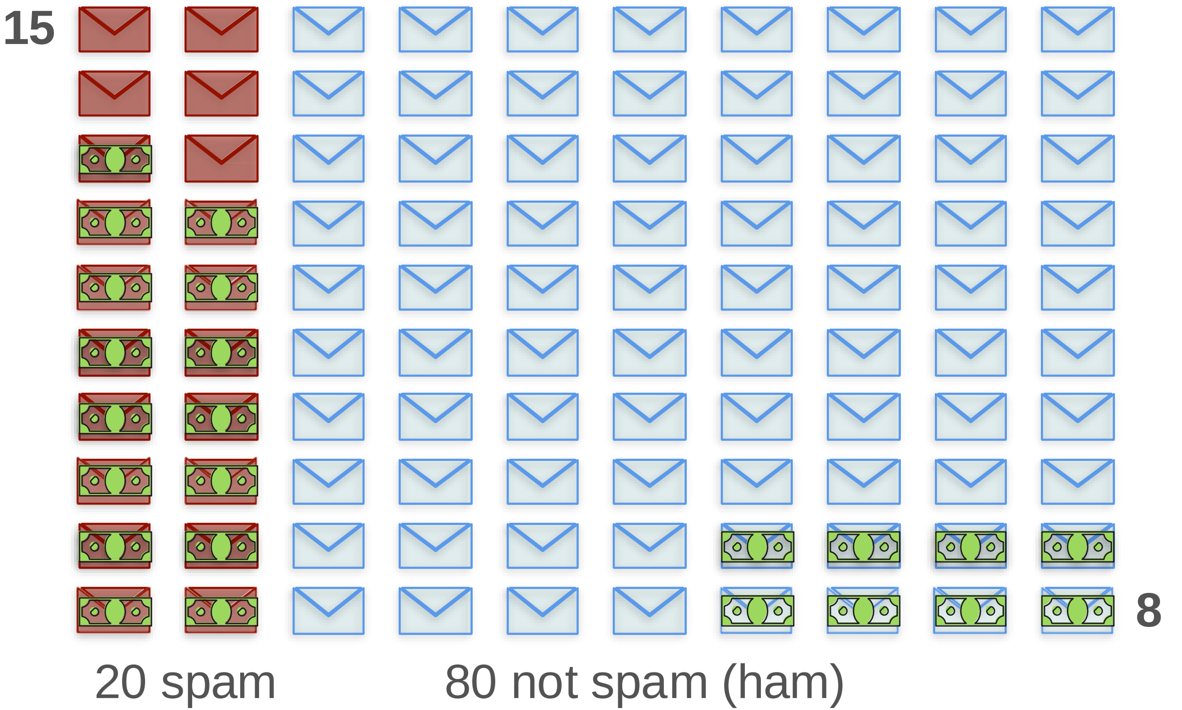
Naive Bayes: Spam Örneği

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{ham}) = \frac{80}{100} = 0.8$$

$$\mathbf{P}(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$

$$\mathbf{P}(\text{winning} \mid \text{ham}) = \frac{8}{80} = 0.1$$



Naive Bayes: Spam Örneği

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{not spam}) = 0.8$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{lottery} \mid \text{ham}) = 0.125$

$\mathbf{P}(\text{winning} \mid \text{spam}) = 0.75$

$\mathbf{P}(\text{winning} \mid \text{ham}) = 0.1$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery} \mid \text{ham}) \cdot \mathbf{P}(\text{winning} \mid \text{ham})}$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)} = 0.913$$

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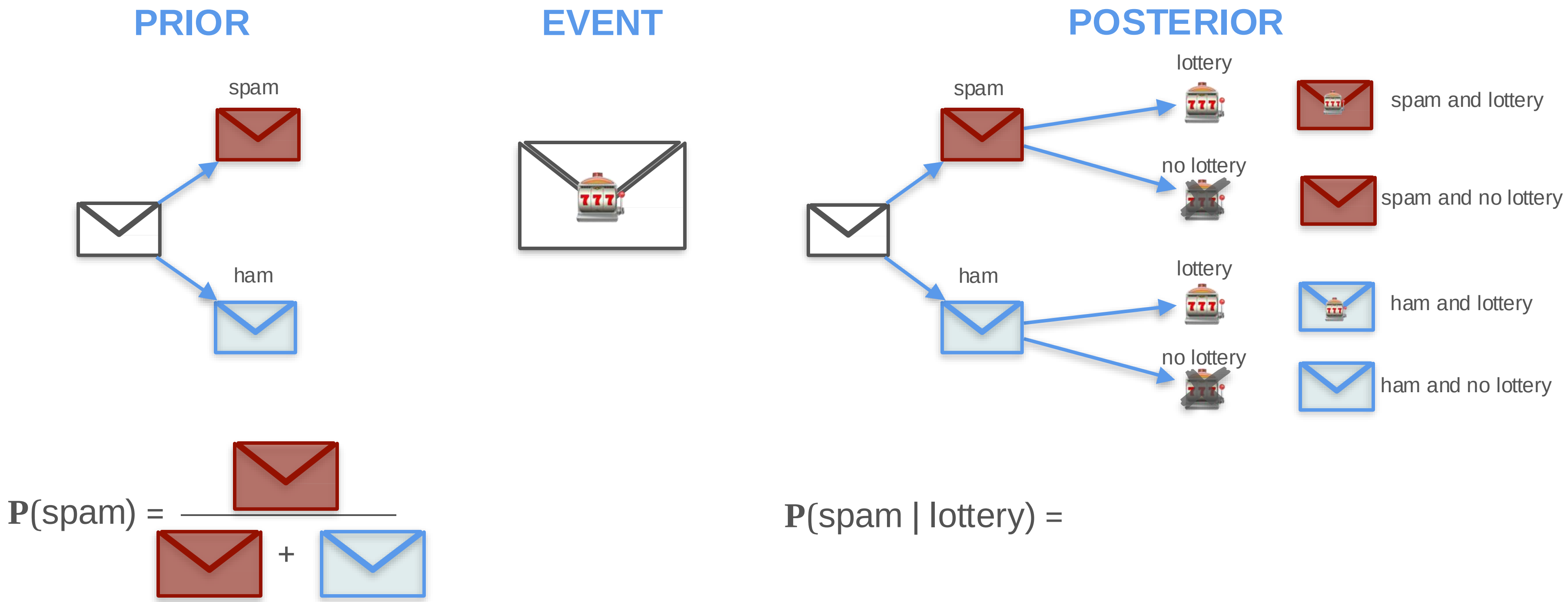
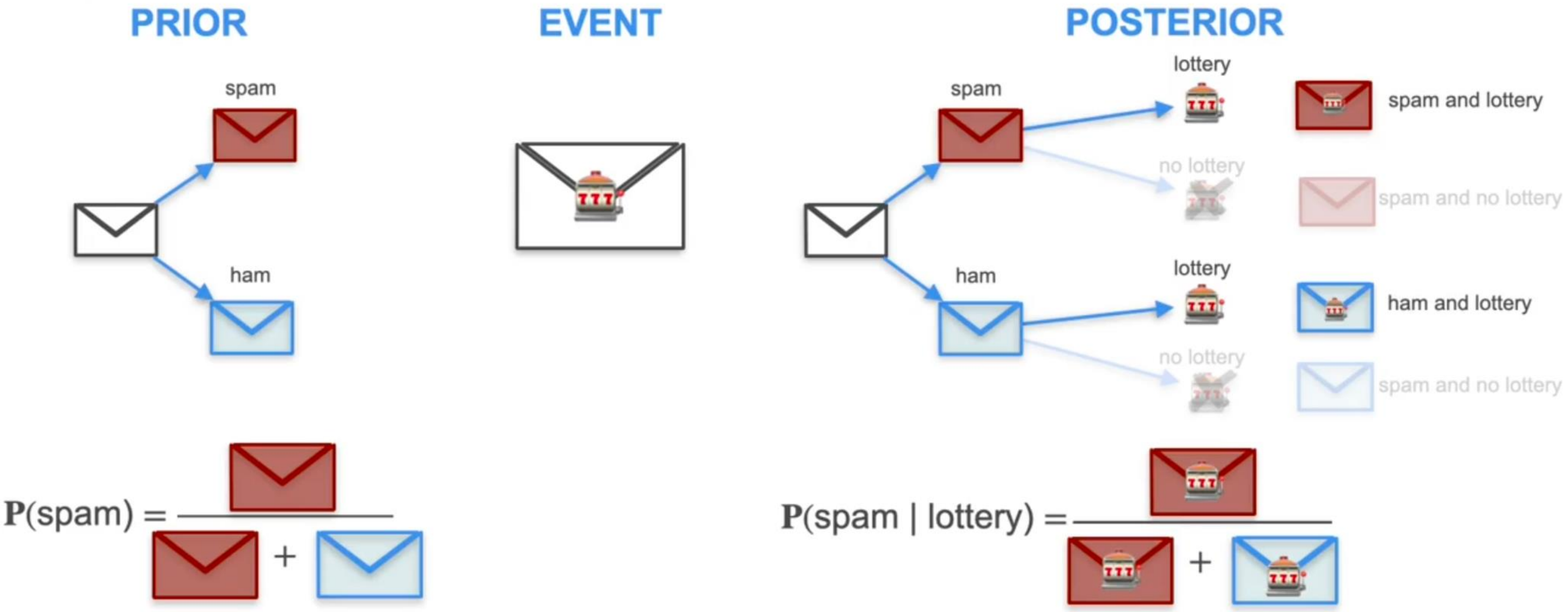
Makine Öğrenmesinde Olasılık

Bayes Theorem



Bayes Theorem



Örnek

Image recognition (Resim Tanıma)

- Resimde bir kedi olma olasılığı nedir?
- $\mathbf{P}(\text{cat} \mid \text{image}) = \mathbf{P}(\text{cat} \mid \text{pixel}_1, \text{pixel}_2, \ldots, \text{pixel}_n)$



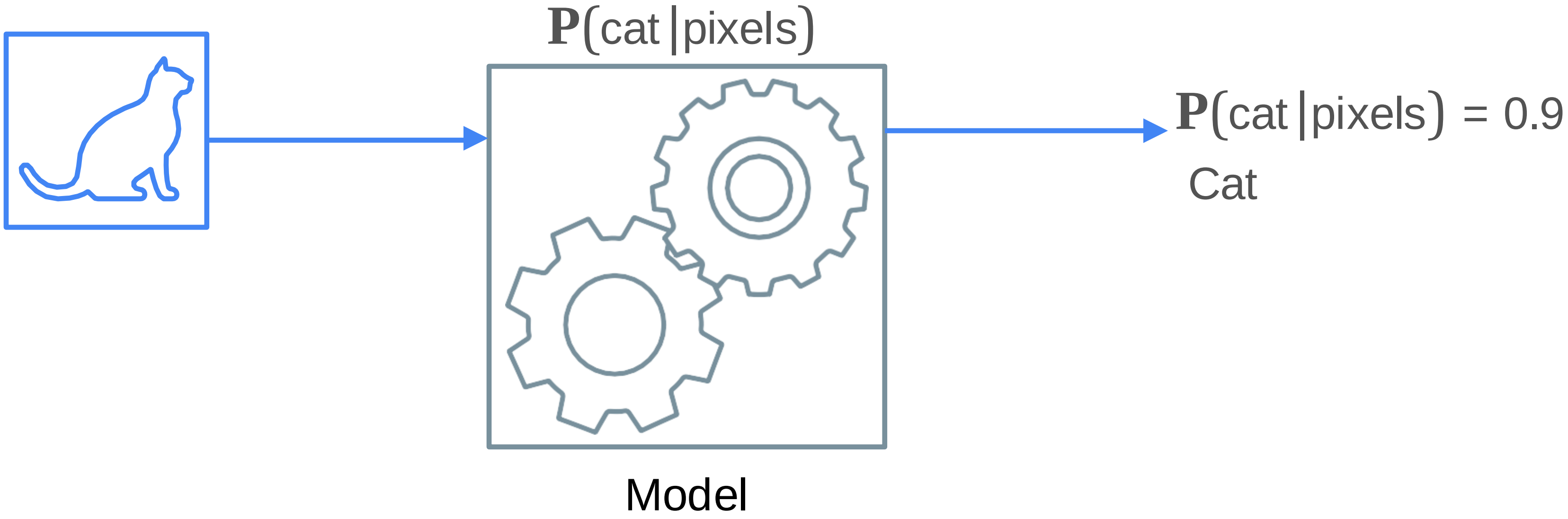
Örnek

Classification (sınıflandırma)

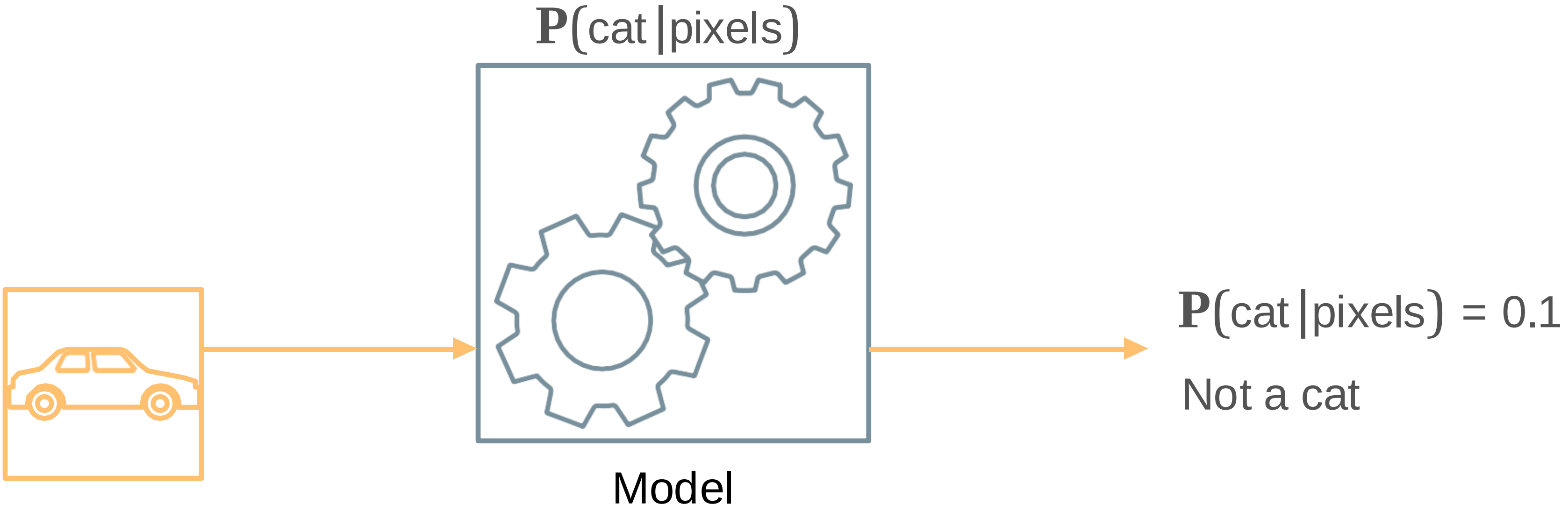
Patient 1	
Age	29
Gender	Female
Height	169 cm
Weight	62 kg
Smoker	No
...	...
Heart rate	63
Blood pressure	120 90

- Kişi sağlıklı mı?
- Hesapla $\mathbf{P}(\text{healthy} \mid \text{symptoms and history})$
- Hesapla $\mathbf{P}(\text{sağlıklı} \mid \text{semptomlar ve hikaye})$

Örnek



Örnek



Özet

Bayes Teoremi

$P(A|B)$ ve $P(B|A)$ koşullu olasılıkları aşağıdaki gibi yazılabilir.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad \text{I}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \qquad \text{II}$$

!!!

$(A \cap B) = (B \cap A)$ olduğundan

Bayes Teoremi

$$P(A|B).P(B) = P(B|A).P(A)$$

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

veya

$$P(B|A) = \frac{P(A|B).P(B)}{P(A)}$$

Bayes Teoremi

Bayes Teoremi

Bir koşullu olasılığı hesaplamak bazen olayla verilen olayı yer değiştirmek işlemleri kolaylaştırır.

Genel Anlamda: Sebep → Sonuç

Sebep: doğrudan gözlemleyemiyoruz

Sonuç: doğrudan gözlemliyoruz

$P(Sonuç \mid Sebep)$ Matematiksel Modelimiz (Hesaplamsı Kolay)

$P(Sebep \mid Sonuç)$ Sonuç çıkarma modelimizi kullanarak bu olasılığı hesaplayabiliriz

Bayes Teoremi

$$P(Sebep \mid Sonuç) = \frac{P(Sonuç \mid Sebep) \, . \, P(Sebep)}{P(Sonuç)}$$

$P(Sebep): sebeple\ ilgili\ önsel\ olasılık$ (prior)

$P(Sebep|Sonuç): sebeple\ ilgili\ sonsal\ olasılık$ (posterior)

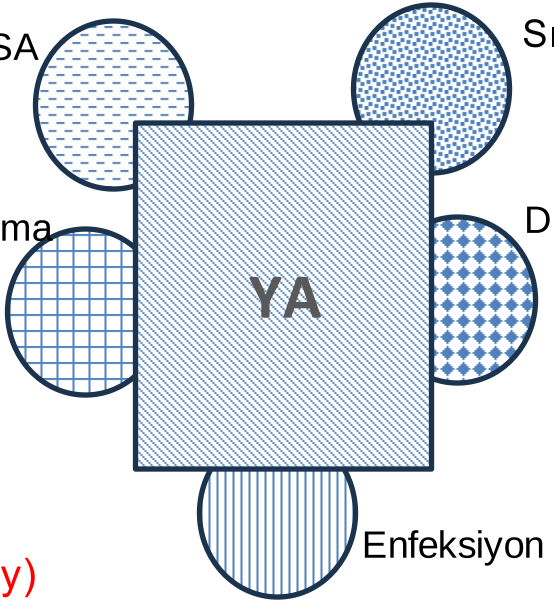
Bayes Teoremi

Örnek: Soğuk Algınlığı → Yüksek Ateş

(Hastalık) (Belirti)

Sebep — doğrudan gözlemleyemiyoruz

Sonuç — doğrudan gözlemliyoruz



$P(Sonuç \mid Sebep)$ Matematiksel MODEL (Hesaplamsı Kolay)

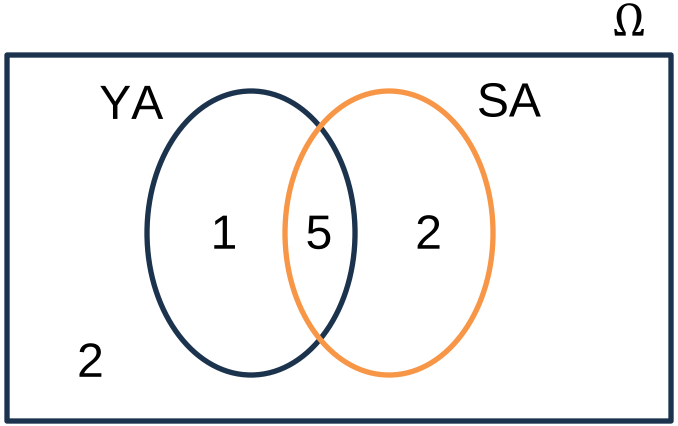
$P(Yüksek\ Ateş \mid Soğuk\ Algınlığı)$ Matematiksel MODEL (Hesaplamsı Kolay)

belirti hastalık

$P(Sebep \mid Sonuç)$ Sonuç çıkarma modelimizi kullanarak bu olasılığı hesaplayabiliriz

$P(Soğuk\ Algınlığı \mid Yüksek\ Ateş) = ?$ Sonuç çıkarma modelimizi kullanarak bu olasılığı hesaplayabiliriz

Hasta	Soğuk Algınlığı	Yüksek Ateş
1	+	+
2	+	+
3	+	+
4	-	+
5	+	-
6	+	+
7	-	-
8	-	-
9	+	+
10	+	-



$$P(soğuk\ algınlığı) = \frac{7}{10}$$

$$P(yüksek\ ateş) = \frac{6}{10}$$

$$P(yüksek\ ateş \mid soğuk\ algınlığı) = \frac{5}{7}$$

$$P(soğuk\ algınlığı \mid yüksek\ ateş) = \frac{5}{6}$$

Birinci Çözüm

$$P(soğuk\ algınlığı) = \frac{7}{10}$$

$$P(yüksek\ ateş) = \frac{6}{10}$$

$$P(yüksek\ ateş \mid soğuk\ algınlığı) = \frac{5}{7}$$

$$P(soğuk\ algınlığı \mid yüksek\ ateş) = \frac{5}{6}$$

$$P(SA \mid YA) = \frac{P(YA \mid SA).P(SA)}{P(YA)}$$

$$\frac{5}{6} = \frac{\frac{5}{7} \cdot \frac{7}{10}}{\frac{6}{10}}$$

İkinci Çözüm

$$P(\text{soğuk algınlığı}) = \frac{7}{10}$$

$$P(\text{yüksek ateş}) = \frac{6}{10}$$

$$P(\text{yüksek ateş} \mid \text{soğuk algınlığı}) = \frac{5}{7}$$

$$P(\text{soğuk algınlığı} \mid \text{yüksek ateş}) = \frac{5}{6}$$

$$P(YA \mid SA) = \frac{P(YA \cap SA)}{P(SA)} = \frac{\frac{5}{10}}{\frac{7}{10}} = \frac{5}{7}$$

$$P(SA \mid YA) = \frac{P(SA \cap YA)}{P(YA)} = \frac{\frac{5}{10}}{\frac{6}{10}} = \frac{5}{6}$$

(zor)

↔

$$P(SA \mid YA) = \frac{\overbrace{P(YA \mid SA).P(SA)}^{P(SA \cap YA)}}{P(YA)} = \frac{\frac{5}{7} \cdot \frac{7}{10}}{\frac{6}{10}} = \frac{5}{6}$$

(zor)

Marjinal ve Koşul Olasılık

Marjinal olasılık, birden fazla olayın gerçekleştiği bir durumda diğer olaylar dikkate alınmayıp sadece bir olay için bulunan olasılığa marjinal olasılık denir.

Bir A olayının marjinal olasılığı P(A) ile gösterilir.

Örnek

200 öğrencinin cinsiyet ve istatistik dersindeki başarı durumlarına göre dağılımı aşağıda verilmiştir.

Cinsiyet	Başarı durumu: Başarılı	Başarı durumu: Başarısız	Toplam
Erkek	30	50	80
Kadın	80	40	120
Toplam	110	90	200

a. Rastgele seçilen bir erkek öğrencinin marjinal olasılığını bulunuz?

b. Rastgele seçilen bir kadın öğrencinin marjinal olasılığını bulunuz?

c. Rastgele seçilen bir öğrencinin başarılı olmasının marjinal olasılığını bulunuz?

d. Rastgele seçilen bir öğrencinin başarısız olmasının marjinal olasılığını bulunuz?

Örnek

200 öğrencinin cinsiyet ve istatistik dersindeki başarı durumlarına göre dağılımı aşağıda verilmiştir.

Cinsiyet	Başarı durumu		Toplam
	Başarılı	Başarısız	
Erkek	30	50	80
Kadın	80	40	120
Toplam	110	90	200

e. Rastgele seçilen bir erkek öğrencinin başarılı olması olasılığı? $P(Başarılı|Erkek)$

f. Rastgele seçilen bir kadın öğrencinin başarılı olması olasılığı? $P(Başarılı|Kadın)$

g. Rastgele seçilen bir erkek öğrencinin başarısız olması olasılığı? $P(Başarısız|Erkek)$

Çözüm-1

Çözüm-2 (Olasılık Ağacı)

Çözüm-3

$$P(B) = P(A).P(B|A) + P(A^c).P\left(B\middle|A^c\right)$$

$$P(Başarılı) = \frac{110}{200}$$

$$P(Başarılı) \stackrel{?}{=} P(E).P(Başarılı|E) + P(K).P(Başarılı|K)$$

Toplam Olasılık Teorisi

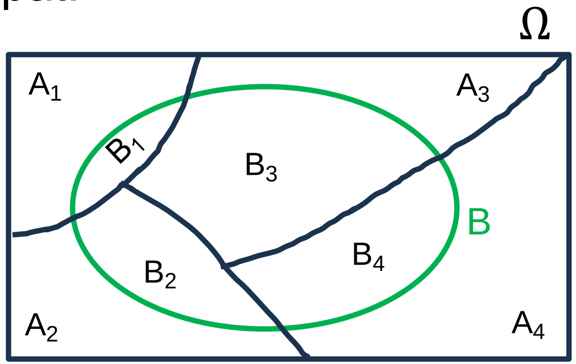
A_1, A_2, … A_n olayları Ω örnek uzayının bölümleri olsun. (Karşılıklı ayrık ve birlikte kapsayıcı) $(A_i \cap A_j = \emptyset)$

B olayı örnek uzayında tanımlı başka bir olay olsun bu durumda B olayının olasılığı $(A_1 \cup A_2 \cup \ \dots \cup A_n = \Omega)$ aşağıdaki gibi hesaplanabilir.

$$P(B) = \sum_{i=1}^{n} P(A_i).P(B|A_i)$$

Toplam Olasılık Teorisi

İspat:



$$B = B_1 \cup B_2 \cup B_3 \cup B_4$$

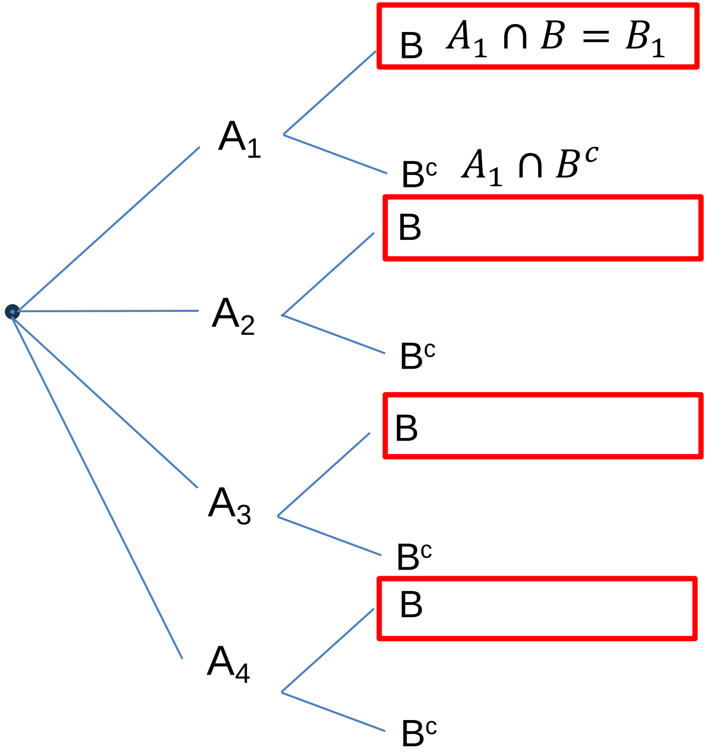
$$B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B) \cup (A_4 \cap B)$$

$$P(B) = P(A_1 \cap B) + \mathrm{P}(A_2 \cap B) + \mathrm{P}(A_3 \cap B) + \mathrm{P}(A_4 \cap B)$$

$$P(B) = P(A_1)P(B|A_1) + \cdots + P(A_4)P(B|A_4)$$

$$P(B) = \sum_{i=1}^{4} P(A_i)P(B|A_i)$$

Toplam Olasılık Teorisi



Toplam Olasılık Teorisi

$P(A_i)$ → A_i bölümü için ön bilgi (önsel olasılık)

B olayı gözlemlendiğinde A_i'nin olasılığı

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

sonsal olasılık

$$= \frac{P(A_i)P(B|A_i)}{P(B)}$$

toplam olasılık teorisi

$$= \frac{P(A_i)P(B|A_i)}{\sum_{j=1}^{n} P(A_i)P(B|A_i)}$$

BAYES TEORİSİ

Genel Bayes Teoremi

Şayet A_1, A_2, … A_n olayları örnek uzayının bölümleri olsun. Örnek uzayında tanımlı herhangi bir B olayı verildiğinde uzayın herhangi bir bölümünün olasılığı aşağıdaki şekilde bulunabilir

$$P(A_i \mid B) = \frac{P(B \mid A_i).P(A_i)}{P(B)}$$

$$= \frac{P(B \mid A_i).P(A_i)}{\sum_{i=1}^{n} P(B \mid A_i).P(A_i)}$$

Örnek

Elektirik malzemelerinin olduğu 4 kutumuz olduğunu kabul edelim

- 1. Kutuda 2000 malzemeden %5'i bozuk
- 2. Kutuda 500 malzemeden %40'i bozuk
- 3. Kutuda 1000 malzemeden %10'i bozuk
- 4. Kutuda 1000 malzemeden %10'i bozuk

Rasgele çektiğşmiz kutudan bir malzeme aldığımızda bunun bozuk olma olasılığı nedir?

Çözüm

Örnek

Önceki örnekte çekilen malzeme bozuk ise bunun 2.kutundan çekilmiş olma olasılığı nedir?

Çözüm

Ödev

Aynı marka aracın üç farklı yakıt türüne göre kat ettiği yol ve motorun vermiş olduğu arıza oranları belirli bir mesafede incelenmiştir. Buna göre benzinli araç yolun %60'nı, dizel araç yolun %30'nu ve sıvılaştırılmış petrol gazı (LPG) ile %10'ununu gittiği gözlenmiştir. Motor arıza oranları ise %2 benzinli araç, %3 dizel araç ve %5 sıvılaştırılmış petrol gazı olduğu belirlenmiştir. Buna göre bu araçlardan rastgele alan bir müşterinin arızalı olma olasılığını bulunuz?